Chapter 8 Sampling Distributions

8.1 Distribution of the Sample Mean

Objectives

1. Describe the distribution of the sample mean: normal population
2. Describe the distribution of the sample mean: nonnormal population

Statistics such as the sample mean, $\bar{x}$, are random variables because their value varies from sample to sample. Therefore, they have a probability distribution. In this chapter, we focus on the shape, center, and spread of statistics such as the sample mean.

The **sampling distribution** of a statistic is a probability distribution for all possible values of the statistic computed from a sample of size *n*.

The **sampling distribution of the sample mean** $\bar{x}$ is the probability distribution of all possible values of the random variable $\bar{x}$ computed from a sample of size *n* from a population with mean μ and standard deviation σ.

❶ Describe the Distribution of the Sample Mean: Normal Population

Activity: Sampling Distribution of the Sample Mean: Normal Population

Go to StatCrunch.com and open the data set "HomeRuns2017". This data set represents all home runs hit during the 2017 baseball season the traveled over the outfield wall (the data set excludes "inside the park" home runs). Because this represents all home runs hit, the data is population data. The variable "Distance" represents the actual distance the ball traveled (measured in feet).

(a) Determine the population mean and standard deviation distance traveled.

(b) Obtain 1000 independent simple random samples of size $n = 4$ for the variable "Distance" from this population. For each sample, determine the sample mean distance traveled.

(c) Draw a histogram of the 1000 sample means. Describe the shape of the distribution.

(d) Determine the mean and standard deviation of the 1000 sample means.

(e) Repeat parts (b) thru (d) for a sample of size $n = 16$. What is the role, if any, of the sample size?

The Mean and Standard Deviation of the Sampling Distribution of $\bar{x}$

Suppose that a simple random sample of size n is drawn from a population* with mean μ and standard deviation σ. The sampling distribution of $\bar{x}$ has mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$. The standard deviation of the sampling distribution of $\bar{x}$, $\sigma_{\bar{x}}$, is called the **standard error of the mean**.

The Shape of the Sampling Distribution of $\bar{x}$ If X Is Normal

If a random variable X is normally distributed, the sampling distribution of the sample mean, $\bar{x}$, is normally distributed.

EXAMPLE Describing the Distribution of the Sample Mean

The distance traditional home runs traveled in 201 is approximately normally distributed with mean 400.3 feet and standard deviation 25.6 feet. What is the probability that a simple random sample of size n = 16 results in a sample mean that is greater than 405 feet? That is, compute $P\left(\bar{x} > 405\right)$.

❷ Describe the Distribution of the Sample Mean: Nonnormal Population

Activity: Sampling Distribution of the Sample Mean: Non-Normal Populations

(a) Go to StatCrunch.com. Select Applets > Sampling distributions. Select the "Uniform" radio button and click Compute!. Or, go to www.pearsonhighered.com/sullivanstats and load the Sampling Distribution Uniform Applet. Note the population mean and standard deviation of the parent population. Obtain 1000 random samples of size n = 4. Note the mean and standard deviation of the 1000 sample means, and the shape of the distribution of the 1000 sample means.

(b) Repeat part (a) for samples of size $n = 10$ and $n = 20$.

(c) On the same applet, draw a skewed distribution by holding the left mouse button down and dragging the mouse over the distribution. Note the population mean and standard deviation of the parent population. Obtain 1000 random samples of size $n = 4$. Note the mean and standard deviation of the 1000 sample means, and the shape of the distribution of the 1000 sample means.

(d) Repeat part (c) for samples of size $n = 10$, $n = 20$, and $n = 30$.

1. The mean of the sampling distribution of the sample mean is equal to the mean of the underlying population, and the standard deviation of the sampling distribution of the sample mean is $\frac{\sigma}{\sqrt{n}}$, regardless of the size of the sample.

2. The shape of the distribution of the sample mean becomes approximately normal as the sample size n increases, regardless of the shape of the underlying population.

We formally state point 2 as the *Central Limit Theorem*.

The Central Limit Theorem

Regardless of the shape of the underlying population, the sampling distribution of $\bar{x}$ becomes approximately normal as the sample size, n, increases.

If the distribution of the population is unknown or not normal, then the distribution of the sample mean is approximately normal provided that the sample size is greater than or equal to 30.

Example Using the Central Limit Theorem

The shape of the distribution of the time spent in a drive-through at a fast-food chain is skewed right. The mean time spent in the drive through is known to be 152.5 seconds with a standard deviation of 30.8 seconds.
(a) To obtain probabilities regarding a sample mean using the normal model, what size sample is required?

(b) The quality-control manager wishes to use a new delivery system designed to get cars through the drive-through system faster. A random sample of 50 cars results in a sample mean time spent at the window of 148.7 seconds. What is the probability of obtaining a sample mean of 148.7 seconds or less, assuming that the population mean is 152.5 seconds? Do you think that the new system is effective?

(c) Treat the next 40 cars that arrive as a simple random sample. There is a 10% chance that the mean time in the drive through will be at or below _______ seconds.

Summary: Shape, Center, and Spread of the Sampling Distribution of $\bar{x}$

	Distribution of the Sample Mean		
Shape, Center, and Spread of the Population	**Shape**	**Center**	**Spread**
Population is normal with mean μ and standard deviation σ	Regardless of the sample size n, the shape of the distribution of the sample mean is normal	$\mu_{\bar{x}} = \mu$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$
Population is not normal with mean μ and standard deviation σ	As the sample size n increases, the distribution of the sample mean becomes approximately normal	$\mu_{\bar{x}} = \mu$	$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

8.2 Distribution of the Sample Proportion

Objectives

❶ Describe the sampling distribution of a sample proportion
❷ Compute probabilities of a sample proportion

❶ Describe the Sampling Distribution of a Sample Proportion

Suppose that a random sample of size n is obtained from a population in which each individual either does or does not have a certain characteristic. The **sample proportion**, denoted $\hat{p}$ (read "p-hat"), is given by

$$\hat{p} = \frac{x}{n}$$

where x is the number of individuals in the sample with the specified characteristic.*
The sample proportion, $\hat{p}$, is a statistic that estimates the population proportion, p.

Example Computing a Sample Proportion

In a survey of 2019 adult Americans aged 18 years or older, 1252 stated that they frequently worry about their financial situation. Find the sample proportion of adult Americans aged 18 years or older who frequently worry about their financial situation.

Activity Describe the Distribution of a Sample Proportion

(a) Go to StatCrunch.com. Select Applets > Sampling distributions. Select the "Binary" radio button and set p (the probability of success) to 0.25. Click Compute!. Or, go to www.pearsonhighered.com/sullivanstats and load the Sampling Distribution Binaary Applet. Note the population mean and standard deviation of the parent population. Obtain 1000 random samples of size $n = 5$. Note the mean and standard deviation of the 1000 sample means, and the shape of the distribution of the 1000 sample means.

(b) Repeat part (a) for samples of size $n = 25$ and $n = 75$.

Sampling Distribution of $\hat{p}$

For a simple random sample of size n with a population proportion p,

- The shape of the sampling distribution of $\hat{p}$ is approximately normal provided $np(1 - p) \geq 10$.
- The mean of the sampling distribution of $\hat{p}$ is $\mu_{\hat{p}} = p$.
- The standard deviation of the sampling distribution of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$.

(c) Describe the sampling distribution of the sample proportion for a population in which the proportion of the population with a certain characteristic is $p = 0.25$ and the sample size is $n = 75$. Compare the results to those in part (b).

❷ Compute Probabilities of a Sample Proportion

Example Compute Probabilities of a Sample Proportion

The proportion of the human population that is left-handed is 0.12. Mensa is an organization of high-IQ individuals (IQs at the 98th percentile or higher). In a random sample of 150 members of Mensa, it was found that 30 were left-handed. What is the probability of obtaining a random sample of 30 or more left-handers in a random sample of 150 individuals assuming the population proportion of left-handers is 0.12? What does this result suggest?